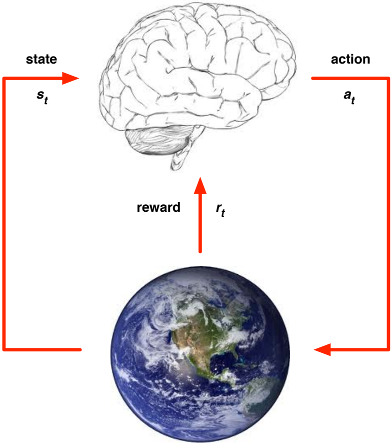
Deep Reinforcement Learning

David Silver, Google DeepMind

Lecture slides - AI4Good Summer Lab - 2019
Slides merged from two different talks/lectures of David Silver

Agent and Environment



- At each step t the agent:
  - Receives state s_t
  - Receives scalar reward r_t
  - Executes action a_t
- The environment:
  - Receives action a_t
  - Emits state s_t
  - Emits scalar reward r_t

Policies and Value Functions

- Policy π is a behaviour function selecting actions given states

$$a = \pi(s)$$

- Value function $Q^\pi(s, a)$ is expected total reward from state s and action a under policy π

$$Q^\pi(s, a) = \mathbb{E}\left[r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + ... \mid s, a\right]$$

"How good is action a in state s?"

Large-Scale Reinforcement Learning

Reinforcement learning can be used to solve *large* problems, e.g.

- Backgammon: 10^{20} states
- Computer Go: 10^{170} states
- Helicopter: continuous state space

Large-Scale Reinforcement Learning

Reinforcement learning can be used to solve *large* problems, e.g.

- Backgammon: 10^{20} states
- Computer Go: 10^{170} states
- Helicopter: continuous state space

How can we scale up the model-free methods for *prediction* and *control* from the last two lectures?

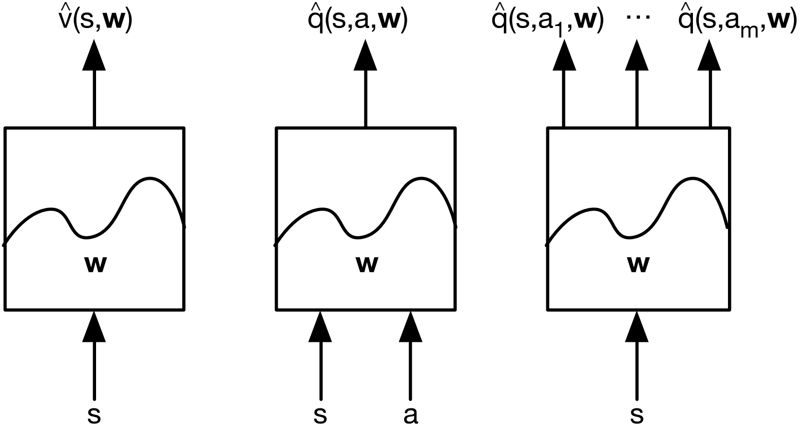
Value Function Approximation

- So far we have represented value function by a *lookup table*
  - Every state s has an entry $V(s)$
  - Or every state-action pair s, a has an entry $Q(s, a)$
- Problem with large MDPs:
  - There are too many states and/or actions to store in memory
  - It is too slow to learn the value of each state individually
- Solution for large MDPs:
  - Estimate value function with *function approximation*

$$\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$$
$$\text{or } \hat{q}(s, a, \mathbf{w}) \approx q_\pi(s, a)$$

  - *Generalise* from seen states to unseen states
  - *Update* parameter $\mathbf{w}$ using MC or TD learning

Types of Value Function Approximation



Which Function Approximator?

There are many function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

Which Function Approximator?

We consider differentiable function approximators, e.g.

- Linear combinations of features
- Neural network
- Decision tree
- Nearest neighbour
- Fourier / wavelet bases
- ...

Furthermore, we require a training method that is suitable for non-stationary, non-iid data

Gradient Descent

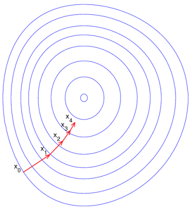
- Let $J(\mathbf{w})$ be a differentiable function of parameter vector $\mathbf{w}$
- Define the *gradient* of $J(\mathbf{w})$ to be

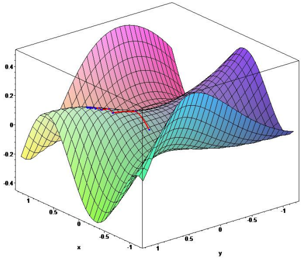
$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \begin{pmatrix} \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_1} \\ \vdots \\ \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_n} \end{pmatrix}$$

- To find a local minimum of $J(\mathbf{w})$
- Adjust $\mathbf{w}$ in direction of -ve gradient

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

where α is a step-size parameter





Action-Value Function Approximation

- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimise mean-squared error between approximate action-value fn $\hat{q}(S, A, \mathbf{w})$ and true action-value fn $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2 \right]$$

- Use stochastic gradient descent to find a local minimum

$$\begin{aligned} -\frac{1}{2} \nabla_{\mathbf{w}} J(\mathbf{w}) &= (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w}) \\ \Delta \mathbf{w} &= \alpha (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w}) \end{aligned}$$

Linear Action-Value Function Approximation

- Represent state *and* action by a *feature vector*

$$\mathbf{x}(S,A) = \begin{pmatrix} \mathbf{x}_1(S,A) \\ \vdots \\ \mathbf{x}_n(S,A) \end{pmatrix}$$

- Represent action-value fn by linear combination of features

$$\hat{q}(S,A,\mathbf{w}) = \mathbf{x}(S,A)^\top \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S,A)\mathbf{w}_j$$

- Stochastic gradient descent update

$$\nabla_{\mathbf{w}} \hat{q}(S,A,\mathbf{w}) = \mathbf{x}(S,A)$$
$$\Delta \mathbf{w} = \alpha(q_\pi(S,A) - \hat{q}(S,A,\mathbf{w}))\mathbf{x}(S,A)$$

Batch Reinforcement Learning

- Gradient descent is simple and appealing
- But it is *not* sample efficient
- Batch methods seek to find the best fitting value function
- Given the agent's experience ("training data")

Least Squares Prediction

- Given value function approximation $\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$
- And *experience* $\mathcal{D}$ consisting of $\langle$*state*, *value*$\rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, ..., \langle s_T, v_T^\pi \rangle\}$$

- Which parameters $\mathbf{w}$ give the *best fitting* value fn $\hat{v}(s, \mathbf{w})$?
- Least squares algorithms find parameter vector $\mathbf{w}$ minimising sum-squared error between $\hat{v}(s_t, \mathbf{w})$ and target values v_t^π,

$$\begin{aligned} LS(\mathbf{w}) &= \sum_{t=1}^{T} (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2 \\ &= \mathbb{E}_{\mathcal{D}} \left[(v^\pi - \hat{v}(s, \mathbf{w}))^2 \right] \end{aligned}$$

Stochastic Gradient Descent with Experience Replay

Given experience consisting of $\langle state, value \rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, ..., \langle s_T, v_T^\pi \rangle\}$$

Repeat:

1. Sample state, value from experience

$$\langle s, v^\pi \rangle \sim \mathcal{D}$$

2. Apply stochastic gradient descent update

$$\Delta \mathbf{w} = \alpha (v^\pi - \hat{v}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(s, \mathbf{w})$$

Stochastic Gradient Descent with Experience Replay

Given experience consisting of $\langle state, value \rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, ..., \langle s_T, v_T^\pi \rangle\}$$

Repeat:

1. Sample state, value from experience

$$\langle s, v^\pi \rangle \sim \mathcal{D}$$

2. Apply stochastic gradient descent update

$$\Delta \mathbf{w} = \alpha (v^\pi - \hat{v}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(s, \mathbf{w})$$

Converges to least squares solution

$$\mathbf{w}^\pi = \underset{\mathbf{w}}{\operatorname{argmin}}\ LS(\mathbf{w})$$

Deep Reinforcement Learning

- Can we apply deep learning to RL?
- Use deep network to represent value function / policy / model
- Optimise value function / policy /model end-to-end
- Using stochastic gradient descent

Bellman Equation

- Value function can be unrolled recursively

$$
\begin{aligned}
Q^{\pi}(s, a) &= \mathbb{E}\left[r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \ldots \mid s, a\right] \\
&= \mathbb{E}_{s'}\left[r + \gamma Q^{\pi}(s', a') \mid s, a\right]
\end{aligned}
$$

- Optimal value function $Q^*(s, a)$ can be unrolled recursively

$$
Q^*(s, a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a\right]
$$

- Value iteration algorithms solve the Bellman equation

$$
Q_{i+1}(s, a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q_i(s', a') \mid s, a\right]
$$

Experience Replay in Deep Q-Networks (DQN)

DQN uses experience replay and fixed Q-targets

- Take action a_t according to ϵ-greedy policy
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
- Sample random mini-batch of transitions (s, a, r, s') from $\mathcal{D}$
- Compute Q-learning targets w.r.t. old, fixed parameters w^-
- Optimise MSE between Q-network and Q-learning targets

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i} \left[\left(r + \gamma \max_{a'} Q(s', a'; w_i^-) - Q(s, a; w_i) \right)^2 \right]$$

- Using variant of stochastic gradient descent

Deep Q-Learning

- Represent value function by deep Q-network with weights w

$$Q(s, a, w) \approx Q^{\pi}(s, a)$$

- Define objective function by mean-squared error in Q-values

$$\mathcal{L}(w) = \mathbb{E}\left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a', w)}_{\text{target}} - Q(s, a, w)\right)^2\right]$$

- Leading to the following Q-learning gradient

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E}\left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w)\right) \frac{\partial Q(s, a, w)}{\partial w}\right]$$

- Optimise objective end-to-end by SGD, using $\frac{\partial L(w)}{\partial w}$

Stability Issues with Deep RL

Naive Q-learning oscillates or diverges with neural nets

1. Data is sequential
   - Successive samples are correlated, non-iid
2. Policy changes rapidly with slight changes to Q-values
   - Policy may oscillate
   - Distribution of data can swing from one extreme to another
3. Scale of rewards and Q-values is unknown
   - Naive Q-learning gradients can be large
     unstable when backpropagated

Deep Q-Networks

DQN provides a stable solution to deep value-based RL

1. Use experience replay
   - Break correlations in data, bring us back to iid setting
   - Learn from all past policies
2. Freeze target Q-network
   - Avoid oscillations
   - Break correlations between Q-network and target
3. Clip rewards or normalize network adaptively to sensible range
   - Robust gradients

Stable Deep RL (1): Experience Replay

To remove correlations, build data-set from agent's own experience

- Take action a_t according to ϵ-greedy policy
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
- Sample random mini-batch of transitions (s, a, r, s') from $\mathcal{D}$
- Optimise MSE between Q-network and Q-learning targets, e.g.

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

Stable Deep RL (2): Fixed Target Q-Network

To avoid oscillations, fix parameters used in Q-learning target

- Compute Q-learning targets w.r.t. old, fixed parameters w^-

$$r + \gamma \max_{a'} Q(s', a', w^-)$$

- Optimise MSE between Q-network and Q-learning targets

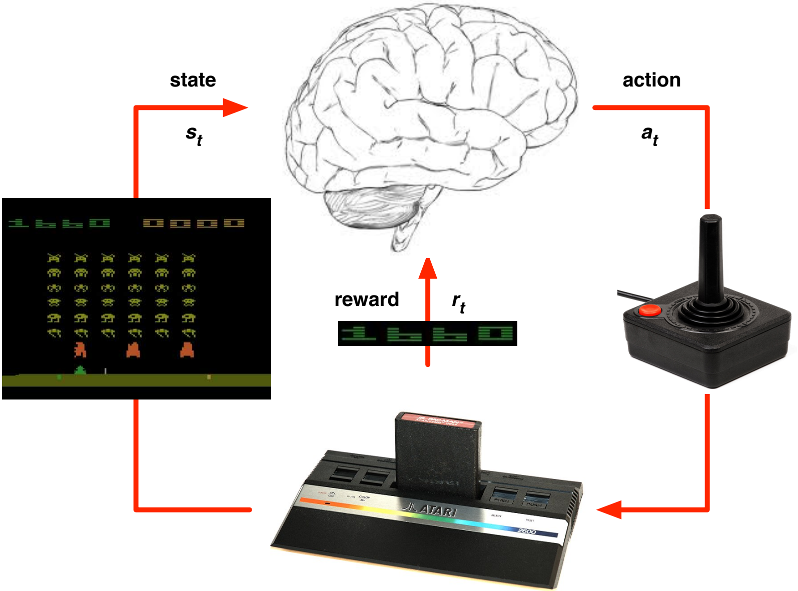
$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left[\left(r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- Periodically update fixed parameters $w^- \leftarrow w$

Stable Deep RL (3): Reward/Value Range

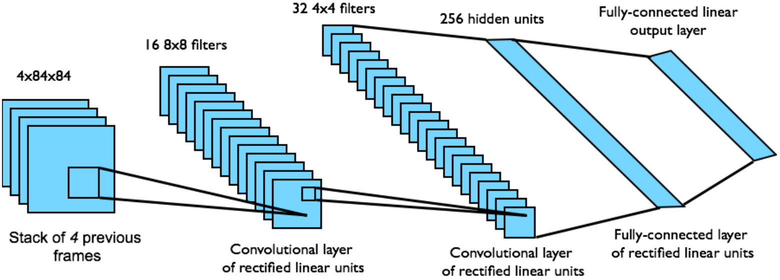
- DQN clips the rewards to $[-1, +1]$
- This prevents Q-values from becoming too large
- Ensures gradients are well-conditioned
- Can't tell difference between small and large rewards

Reinforcement Learning in Atari



DQN in Atari

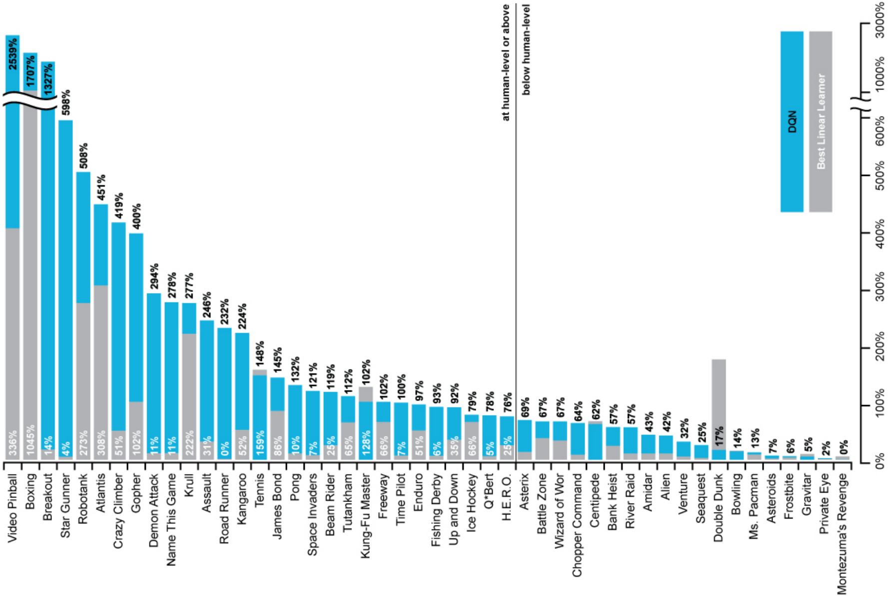
- End-to-end learning of values $Q(s, a)$ from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is $Q(s, a)$ for 18 joystick/button positions
- Reward is change in score for that step



Network architecture and hyperparameters fixed across all games

[Mnih et al.]

DQN Results in Atari



How much does DQN help?

DQN

	Q-learning	Q-learning + Target Q	Q-learning + Replay	Q-learning + Replay + Target Q
Breakout	3	10	241	**317**
Enduro	29	142	831	**1006**
River Raid	1453	2868	4103	**7447**
Seaquest	276	1003	823	**2894**
Space Invaders	302	373	826	**1089**

Conclusion

- RL provides a general-purpose framework for AI
- RL problems can be solved by end-to-end deep learning
- A single agent can now solve many challenging tasks
- Reinforcement learning $+$ deep learning $=$ AI

Questions?

"The only stupid question is the one you never ask" -*Rich Sutton*